Comparison of Simple Turbulent Heating Estimates for Lifting Entry Vehicles

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Nomenclature

c_f = skin-friction coefficient, $2\tau_w/\rho_1 u_1^2$
$F_c, F_{Rx}, F_{R\theta}$ = multipliers that transform the skin friction, length Reynolds number and momentum thickness Reynolds number, respectively, from the compressible to incompressible flow
h = enthalpy
q_w = wall heat flux
Re_x = Reynolds number, $\rho_1 u_1 x/\mu_1$
St = Stanton number, $q_w/\rho_1 u_1(h_{aw} - h_w)$
u = velocity
x = length from leading edge
α = angle of attack
ρ = density
μ = viscosity
τ_w = wall shear
θ = momentum thickness
σ = Prandtl number

Subscripts

1 = boundary-layer edge conditions
aw = adiabatic wall
w = wall
x = based on length
0 = stagnation conditions
θ = based on momentum thickness
s = stagnation enthalpy at the local pressure
exp = experimental value
calc = calculated value

Superscript

* = reference conditions: $Re_x^* = \rho^* u_1 x/\mu^*$, $c_f^* = 2\tau_w/\rho^* u_1^2$, $St^* = q_w/\rho^* u_1(h_{aw} - h_w)$

I. Introduction

AERODYNAMIC heating is an important factor in the design of lifting vehicles which enter the Earth's atmosphere from orbit. In particular, the feasibility of developing a large vehicle that can be flown many times without extensive refurbishment depends strongly on the highest temperatures attained by the heat shield. The most important region in the design of a reuseable shield is the bottom (windward) surface, which is about 40% of the total vehicle surface area. The highest temperatures on this surface, outside of the regions near stagnation points or leading edges, can occur when the boundary layer is turbulent. Accurate predictions of heating rates for a turbulent boundary layer are therefore of importance in the design of these vehicles. In preliminary analyses, where heating calculations are made many times in conjunction with trajectory calculations and elaborate computational schemes are not warranted, the methods used most often are generally valid only for a flat plate.

It is the purpose of this Note to compare the heat-transfer predictions from four simple calculation methods with some experimental data for a turbulent boundary layer on a flat plate in order to determine which of the estimates is the most accurate. The four methods considered are 1) Eckert's reference enthalpy, 2) Spalding and Chi's correlation, 3) the reference density-viscosity product ($\rho\mu$), and 4) the adiabatic wall reference enthalpy. In addition, a sample calculation of the highest bottom centerline radiation equilibrium surface temperatures for fully turbulent flow on a typical lifting entry vehicle is given for each of the methods to illustrate the differences between the predictions.

Configurations proposed for large lifting entry vehicles are delta-like in planform and have little longitudinal or transverse curvature in the region where the boundary layer is turbulent. In addition, this region is sufficiently far from the nose and leading edges that bluntness effects can be ignored. More importantly, it is believed that at the centerline, the effects of the three-dimensional character of a turbulent boundary-layer flow result in only small additions to the heating predicted for purely two-dimensional flow. All of these simplifications suggest that the heating is like that to a flat plate in a uniform flow. The remaining effects which should be accounted for are those due to the compressibility of the fluid and the variation of transport properties through the boundary layer.

Comparisons like the one given here have been made previously[1,2] and the conclusions have been that Spalding and Chi's correlation for the skin friction when used with Von Kármán's equation for the Reynolds analogy factor gives the best estimate of the heat transfer. These comparisons have been rather general, however, and have not included the adiabatic wall reference enthalpy method. The present comparison is concerned with a limited range of boundary-layer edge conditions for which the local Mach number is between 1 and 9 and the length Reynolds number is between 10^6 and 10^7 at peak heating in fully turbulent flow for angles of attack between 10° and 60°. The applicable range of the ratio of the wall to the adiabatic wall enthalpy is between 0.06 and 0.15. The consideration of such a restricted set of boundary-layer edge conditions and wall enthalpy ratios is intended to make this comparison applicable to a large lifting entry vehicle with a radiatively cooled heat shield.

II. Comparison

Each of the methods compared here uses the same procedure to account for the variable density and transport properties in a compressible, turbulent boundary layer. The real flow is transformed into an equivalent incompressible flow for which the density and transport properties are uniform and are evaluated at a reference enthalpy. Well-established laws for skin friction and heat transfer in low-speed flows are then used to solve the problem in the equivalent incompressible flow and, finally, the solutions are transformed back to the real, compressible flow. The quantities in the transformed flow with uniform properties (denoted by the superscript *) are defined by $c_f^* = F_c c_f$, $St^* = F_c St$, $Re_x^* = F_{Rx} Re_x$, and $Re_\theta^* = F_{R\theta} Re_\theta$, where $F_{R\theta} = F_c F_{Rx}$. A summary of these multiplication factors and the reference enthalpies are given in Table 1, along with the skin-friction law and Reynolds analogy factor used for each method.

A completely consistent comparison would use the same skin friction law and analogy factor for each method. (At the larger Reynolds numbers, there can be differences of as much as 15% in the skin-friction predictions and 20% in the Reynolds analogy factor.) However, each method has been proposed and developed with a particular skin-friction law and analogy factor, and the refinement of using the same law for each is not made here.

In the subsequent comparison the effect of the wall enthalpy ratio, h_w/h_{aw}, is considered only for small values applicable to a lifting entry vehicle. The correct dependence of the heating on the Reynolds number is adequately accounted for in the skin-friction laws. Only data with wall enthalpy ratios of 0.143 or less are considered; this limits the number of applicable experiments to only two. One is the data for fully turbu-

Received July 8, 1970; revision received August 7, 1970.
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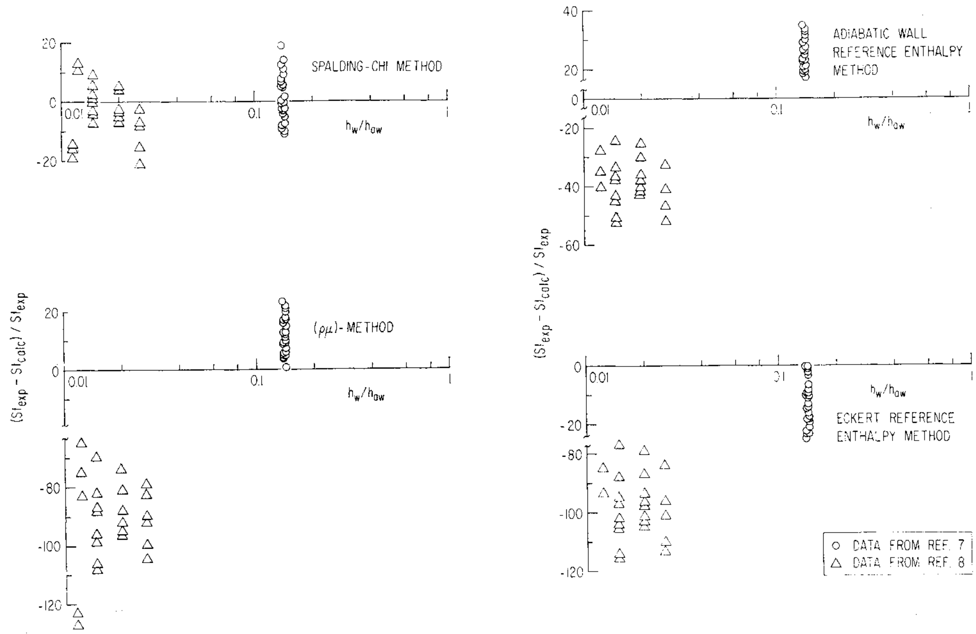

Fig. 1 Comparison of measured and calculated Stanton numbers for a turbulent boundary layer (data from Refs. 7 and 8).

lent flow on a flat plate given in the report by Wallace and McLaughlin.[7] The other is that given by Hopkins and Nerem[8] for a tripped turbulent boundary layer near the entrance in a hollow cylinder exposed to supersonic flow in a shock tube. The latter data therefore include some effect of transverse curvature, but the experiment was designed to simulate a flat plate. A summary of the conditions for these two experiments is given in Table 2.

Comparison of the data with the predictions from each method is made in the form $(St_{\text{exp}} - St_{\text{calc}})/St_{\text{exp}}$ and is shown in Fig. 1. Only the low wall enthalpy ratios are considered here because the comparison for higher values has been adequately discussed in Refs. 1, 2, and 4. It can be concluded from the comparison made here that Spalding and Chi's correlation with Von Kármán's equation for the Reynolds analogy factor gives the best agreement with data. The agreement of Spalding and Chi's correlation with this data strongly suggests that it is the only method of the four tested that correctly accounts for the dependence of the heating on the wall enthalpy ratios applicable to entry of a lifting body with a radiatively cooled heat shield.

At the highest wall enthalpy ratio used here, the predictions from Eckert's method are generally too large and those from the $(\rho\mu)$ method and the adiabatic wall reference enthalpy method are too small. At the lower wall enthalpy ratios, each of these three methods overpredicts the heat transfer. The predictions could be improved by using a different effective Reynolds analogy factor for each method. However, direct measurements of $2St/c_f$ (Ref. 7) indicate values of 1.0–1.12. Predictions from Von Kármán's equation agree with these values while the value given by Colburn's equation is 1.25 (for $\sigma = 0.715$). Although the predictions from Eckert's method would be uniformly improved by using a Reynolds analogy factor of unity, the $(\rho\mu)$ and adiabatic wall reference enthalpy methods would require values greater than that given by Colburn's equation at the higher wall enthalpy ratio and values less than unity at the lower wall enthalpy ratios. Spalding and Chi's correlation is the only method to give uniformly accurate predictions with a Reynolds analogy factor that essentially coincides with measured values. For the data shown here, the error for this correlation with Von Kármán's equation for the Reynolds analogy factor is between $\pm 20\%$. Errors for the other methods are as large as 100% at the lowest wall enthalpy ratios. For this reason, it is suggested that Spalding and Chi's correlation with the Von Kármán equation for the Reynolds analogy factor gives the best estimate of turbulent heat transfer on a flat plate in a uniform stream with boundary-layer edge Mach number, Reynolds number, and wall enthalpy ratios corresponding to those expected on the bottom surface of a lifting entry vehicle at peak turbulent heating.

III. Heating Estimates for Flight Conditions

Each of these four methods is now used to compute the peak temperature on the windward side of a flat, delta wing during entry along an equilibrium glide trajectory in order to illustrate the difference in heating predictions for flight conditions. A 200-ft-long vehicle with an 80° sweep and a

Table 1 Transformation factors, skin-friction laws, and Reynolds analogy factors for the turbulent heating methods

Method (Reference)	F_c	$F_{R\theta}$	h^*	c_f^*	$2St/c_f$
Eckert[3]	ρ_1/ρ^*	μ_1/μ^*	$\frac{1}{2}(h_w + h_1) + 0.22r(h_0 - h_1)$	^{a}	^{b}
Adiabatic wall[4]	ρ_1/ρ^*	μ_1/μ^*	$h_1 + 0.72r(h_0 - h_1)$	^{a}	^{e}
Spalding-Chi[5]	$\left[\int_0^1 \left(\frac{\rho}{\rho_1}\right)^{1/2} d\left(\frac{u}{u_1}\right)\right]^{-2}$	$\frac{(h_{aw}/h_w)^{0.772}}{(h_w/h_1)^{0.702}}$		^{d}	^{b}
$(\rho\mu)$ (Ref. 6)	$\rho_1\mu_s/\rho^*\mu^*$	μ_1/μ_s		^{e}	^{c}

^{a} $c_f^* = 0.0592\, Re_x^{*-1/5}$.
^{b} Von Kármán's equation, $2St^*/c_f^* = (1 + 5\,(c_f^*/2)^{1/2}\{\sigma^* - 1 + \ln[(5\sigma^* + 1)/6]\})^{-1}$.
^{c} Colburn's equation, $2St^*/c_f^* = \sigma^{*-2/3}$.
^{d} Table given in Ref. 5.
^{e} Schultz-Grunow $c_f^* = 0.37[\log(Re_x^* + 3000)]^{-2.584}$.

Table 2 Test conditions for data from Refs. 7 and 8

Reference	M_1	Re_x	h_w/h_{aw}	Configuration
7	5.67–7.4	2.5×10^6–6×10^7	0.139–0.142	Flat plate
8	2.5–3.5	10^5–10^6	0.012–0.027	Hollow cylinder

planform loading of 50 lb/ft² is used. The lift coefficient is taken to be Newtonian ($2 \sin^2\alpha\cos\alpha$). For this particular example, transition onset is assumed to occur at a fixed length Reynolds number of 10^6 and fully turbulent flow at 2×10^6. The point of transition onset is taken as a reasonable estimate for the virtual origin of the turbulent boundary layer. Boundary-layer edge conditions are those behind a plane oblique shock and the effect of three-dimensional flow on the centerline heating is approximated by a small correction to the prediction for two-dimensional flow, which accounts for a transverse velocity gradient. The correction increases nearly linearly with angle of attack and amounts to an increase in heating of less than 13% at 50°.

The results are given in terms of the peak radiation equilibrium temperature (emmissivity of 0.85) on the vehicle centerline as a function of angle of attack and are summarized in Fig. 2. It is emphasized that the altitude at peak heating is different for each angle of attack because the equilibrium glide trajectory is different. The altitude of peak heating increases with increasing angle of attack, while the flight velocity for peak heating in fully turbulent flow is essentially constant at about 17,500 fps. These results are shown only to illustrate the different temperature predictions by each method, so the temperature differences rather than the magnitudes are to be emphasized. Spalding and Chi's correlation, which gives the best agreement with experimental data, gives nearly the lowest peak temperature estimates. The difference between the lowest and highest predictions can be greater than 200°F.

The peak temperatures are given in this example for a specific transition criterion and lift coefficient curve in order to show the differences which are obtained by changing only the method for predicting heating in fully turbulent flow. However, it is known that the peak temperature dependence on angle of attack is also very sensitive to the choice of transition criterion and lift coefficient. These are additional factors which can affect the peak temperature predictions by an amount comparable to that of the different heat-transfer prediction methods.

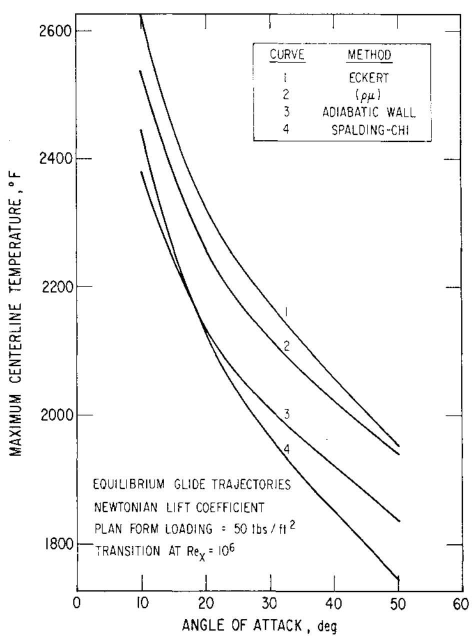

Fig. 2 Maximum centerline radiation equilibrium temperatures (emissivity of 0.85) at peak turbulent heating for a 200-ft, flat delta wing with 80° sweep.

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Effect of Suspension Line Elasticity on Parachute Loads

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Nomenclature

$C_DS(t)$	= instantaneous drag-area of parachute during inflation
C_DS_o	= drag-area of parachute at steady full inflation
$E(t)$	= normalized force, $E = m_2\ddot{x}_2/q_sC_DS_o$
k	= suspension line spring constant (slope of linear approximation to force-elongation curve)
M	= load amplification factor, $M = E_{max}$
m_1,m_2	= mass of parachute and payload, respectively
n	= exponential power, $n = 1, 2, 3, 4$
q_s	= dynamic pressure at time of snatch force, $q_s = \rho[\dot{x}_1(0)]^2/2$
T	= natural oscillation period of system, $T = 2\pi/\omega$
t	= time from snatch force
t_f	= filling time (from snatch force to full inflation)
$x_1(t)$, $x_2(t)$	= displacement of parachute and payload, respectively
$\xi(t)$	= suspension line elongation, $\xi = x_2 - x_1$
ρ	= atmospheric density
φ	= parameter relating the velocities of m_1 and m_2, $\varphi = (\dot{x}_2/\dot{x}_1) - 1$

Received July 16, 1970.

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